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Unitarity constraint for threshold coherent pion photoproduction on the deuteron and chiral perturbation theory

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Abstract

The contribution of the two-step process $\gamma d \to pn \to \pi^0 d$ to the imaginary part of the amplitude for coherent pion production on the deuteron is calculated exploiting unitarity constraints. The result shows that this absorptive process is not negligible and has to be considered in an extraction of the elementary neutron production amplitude from the $\gamma d \to \pi^0 d$ cross section at threshold. In addition, it is argued that a consistent calculation of $\gamma d \to \pi^0 d$ in baryon chiral perturbation theory beyond next-to-leading order requires the inclusion of this absorptive process.

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Recently there has been considerable interest in the coherent electromagnetic production of pions from the deuteron near threshold. The main motivation thereby is to gain information on the elementary neutron amplitude $\gamma n \to \pi^0 n$ which is experimentally not directly accessible. A first measurement of the $ed \to e\pi^0 d$ reaction near $q_\mu^2 = -0.075\,\mathrm{GeV}^2/c^2$ will be performed soon at MAMI [1]. However, it is already known for a long time that at threshold the $\gamma d \to \pi^0 d$ process is dominated by two-nucleon production mechanisms [2]. Therefore, a careful theoretical analysis which allows the separation of the one-nucleon process is essential. Recently, chiral perturbation theory (χPT) in the heavy baryon formulation has been applied to this reaction by Beane et al. [3] predicting a real threshold amplitude. Very recently, this work has been improved and extended beyond next-to-leading order in the chiral power counting scheme [4].

It is the purpose of the present paper to point out that an additional contribution of an absorptive two-step process, $\gamma d \to pn \to \pi^0 d$, has to be included, which leads to a complex amplitude even at threshold. The presence of such a competing deuteron disintegration channel has an analogue in the case of πd elastic scattering, where the contribution of the absorptive process $\pi d \to NN \to \pi d$ is known to be of the order of 10% of the total amplitude (see e.g. [5]). An effect of this size would not be negligible for the electromagnetic reaction due to the relative smallness of the single-nucleon amplitude, one is mainly interested in. In the case of $\pi d \to \pi d$, the imaginary part of the scattering length $a_{\pi d}$ is related to the total absorption cross section through

$$\Im m\, a_{\pi d} = \frac{1}{4\pi} \lim_{p_{\pi d} \to 0} p_{\pi d}\; \sigma(\pi d \to X), \tag{1}$$

where $p_{\pi d}$ is the pion momentum in the c.m. system. This relation follows directly from the optical theorem. An analogous unitarity constraint on the $\gamma d \to \pi^0 d$ amplitude near threshold is present and will be treated as first point below. In view of the fact that the absorptive process has not been considered in [3,4], we will analyze its role within the χPT framework as second point.

As is well known, the unitarity of the S-matrix leads to constraints for the corresponding

reaction amplitudes. For our purposes, it is sufficient to consider coupled two-particle channels where the particles are subject to interactions which are invariant under time-reversal. In this case, following the conventions of Ref. [6], the imaginary part of the partial wave T-matrix element of total angular momentum J for the reaction $a = (a_1 a_2) \to b = (b_1 b_2)$ fulfils the following unitarity constraint

$$\Im m\, T^J_{\lambda_{a_1}\lambda_{a_2}\to\lambda_{b_1}\lambda_{b_2}}(W) = \sum_c p_c \sum_{\lambda_{c_1}\lambda_{c_2}} T^J_{\lambda_{a_1}\lambda_{a_2}\to\lambda_{c_1}\lambda_{c_2}}(W) \left(T^J_{\lambda_{b_1}\lambda_{b_2}\to\lambda_{c_1}\lambda_{c_2}}(W)\right)^* . \qquad (2)$$

Here, λ_{c_i} is the helicity of particle i in the channel c, and p_c denotes the c.m. momentum of the two particles in the channel c. The first sum on the rhs of (2) runs over all open channels c for a given total c.m. energy W. In terms of the T^J the total helicity amplitude for $a \to b$ is given by

$$T_{\lambda_{a_1}\lambda_{a_2}\to\lambda_{b_1}\lambda_{b_2}}(W,\phi,\theta) = 8\pi W \sum_J (2J+1) e^{i(\lambda_a-\lambda_b)\phi} d^J_{\lambda_a\lambda_b}(\theta) T^J_{\lambda_{a_1}\lambda_{a_2}\to\lambda_{b_1}\lambda_{b_2}}(W), \qquad (3)$$

with $\lambda_a = \lambda_{a_1} - \lambda_{a_2}$, $\lambda_b = \lambda_{b_1} - \lambda_{b_2}$, and θ, ϕ are the spherical coordinates of the outgoing particle b_1.

We apply relation (2) to the three coupled channels $\pi^0 d$, pn, and γd at the π^0 production threshold, i.e., in the limit $W \to W_0 \equiv m_{\pi^0} + m_d$, which, of course, implies $p_{\pi^0 d} \to 0$. In this limit, therefore, no contribution from $c = (\pi^0 d)$ to the sum occurs due to the vanishing phase space factor. Moreover, at threshold we can restrict ourselves to partial waves of angular momentum and parity $J^\pi = 1^-$. Thus $\pi^0 d \to pn$ is described by a single matrix element A, with

$$T^1_{\lambda_d\to\lambda_p\lambda_n} = \frac{1}{3}(101\lambda_p - \lambda_n|1\lambda_p - \lambda_n)A. \qquad (4)$$

For $\gamma d \to \pi^0 d$ at threshold, two independent matrix elements remain, $E_{\pi^0 d}$ and $M_{\pi^0 d}$. They correspond to the electric dipole (E1) and magnetic quadrupole (M2) radiation allowed for the $1^+ \to 1^-$ transition of the hadronic system. One finds

$$T^1_{\lambda_\gamma\lambda_d\to\lambda'_d} = \frac{1}{3\sqrt{6}} \sum_{L=1,2} \sqrt{2L+1}\,(1-\lambda_d L\lambda_\gamma|1\lambda_\gamma-\lambda_d)\,(\delta_{L,1}E_{\pi^0 d} + \lambda_\gamma \delta_{L,2} M_{\pi^0 d}). \qquad (5)$$

Finally, for $\gamma d \to {}^3P_1(pn)$ one obtains

$$T^1_{\lambda_\gamma\lambda_d\to\lambda_p\lambda_n} = \frac{1}{3\sqrt{2}}(101\lambda_p - \lambda_n|1\lambda_p - \lambda_n)$$
$$\sum_{L=1,2}\sqrt{2L+1}\,(1-\lambda_d L\lambda_\gamma|1\lambda_\gamma-\lambda_d)\,(\delta_{L,1}E_{pn}+\lambda_\gamma\delta_{L,2}M_{pn}), \tag{6}$$

where E_{pn} and M_{pn} are the corresponding E1 and M2 matrix elements of the disintegration process. The various total cross sections in terms of these matrix elements are

$$\sigma(\pi^0 d \to pn) = \frac{4\pi}{3}\frac{p_{pn}}{p_{\pi^0 d}}|A|^2, \tag{7}$$

$$\sigma(\gamma d \to \pi^0 d) = \frac{4\pi}{6}\frac{p_{\pi^0 d}}{p_{\gamma d}}\left(|E_{\pi^0 d}|^2 + |M_{\pi^0 d}|^2\right), \tag{8}$$

$$\sigma\left(\gamma d \to {}^3P_1(pn); \mathrm{E1{+}M2}\right) = \frac{4\pi}{6}\frac{p_{pn}}{p_{\gamma d}}\left(|E_{pn}|^2 + |M_{pn}|^2\right). \tag{9}$$

Taking now $a = (\gamma d)$ and $b = (\pi^0 d)$, relation (2) leads to

$$\Im m\, E_{\pi^0 d}(W_0) = \frac{1}{\sqrt{3}}\, p_{pn}\, E_{pn}(W_0)\, A^*(W_0), \tag{10}$$

and an analogous relation for $M_{\pi^0 d}$ and M_{pn}, respectively. Since the lhs of (10) is real, relation (10) implies that the phases of the complex matrix elements A and E_{pn} are equal. Indeed, evaluating (2) with $a = (\pi^0 d)$, $b = (pn)$ and $a = (\gamma d)$, $b = (pn)$ provides us with

$$A(W_0) = |A(W_0)|\, \exp\left(i\delta_{^3P_1}(W_0) + in\pi\right), \tag{11}$$

and

$$E_{pn}(W) = |E_{pn}(W)|\, \exp\left(i\delta_{^3P_1}(W) + ik\pi\right), \quad W \le W_0, \tag{12}$$

respectively, where $\delta_{^3P_1}$ is the nucleon-nucleon scattering phase shift in the ${}^3P_1$ channel. Eq. (12) is simply Watson's theorem applied to deuteron photodisintegration and is valid for all energies below the pion production threshold, whereas Eq. (11) is valid for $W = W_0$ only. Finally, we mention that taking $a = b = (\pi d)$ in (2), leads to the constraint for the imaginary part of the πd scattering length in (1).

Using (7) and (9), and the detailed balance relation

$$3\, p_{\pi^0 d}^2\, \sigma(\pi^0 d \to pn) = 4\, p_{pn}^2\, \sigma(pn \to \pi^0 d), \tag{13}$$

our main result (10) can be rewritten as

$$|\Im m\, E_{\pi^0 d}| = \frac{1}{\sqrt{2}\,\pi}\, p_{pn} \sqrt{\frac{p_{\gamma d}}{p_{\pi^0 d}}\, \sigma\left(\gamma d \to {}^3P_1(pn); \mathrm{E1}\right)\, \sigma(pn \to \pi^0 d)}. \tag{14}$$

At this level, there is, however, no way to fix the sign. In order to get a numerical value, we take for the hadronic cross section the experimental result given by Hutcheon et al. [7],

$$\lim_{p_{\pi^0 d} \to 0} 2\, \frac{m_\pi}{p_{\pi^0 d}}\, \sigma(pn \to \pi^0 d) = 184 \pm 5 \pm 13\, \mu b. \tag{15}$$

The partial cross section $\gamma d \to {}^3P_1(pn)$ is at present not available although it could in principle be obtained from a multipole analysis. There are, however, reliable theoretical models available which reproduce all deuteron photodisintegration data in this energy region [8]. At this energy, the E1 matrix element is mainly given by π-exchange current contributions, which can be calculated in a largely model-independent way by taking advantage of gauge-invariance constraints (Siegert's theorem). The underlying nucleon-nucleon interaction and also all model-dependent transverse electromagnetic currents, like the $\Delta(1232)$ excitation current, have little effect on this matrix element. We take the value

$$\sigma\left(\gamma d \to {}^3P_1(pn); \mathrm{E1}\right) = 10.5\, \mu b \tag{16}$$

from an updated version of the model of Ref. [9]. In order to relate our result to the χPT calculations of Beane et al. [4], we switch to their normalization (and notation) of the electric dipole amplitude, $E_d \equiv E_{\pi^0 d}/4$, and obtain

$$|\Im m\, E_d| = 0.22 \times 10^{-3}/m_{\pi^+}. \tag{17}$$

The cross section $\sigma\left(\gamma d \to {}^3P_1(pn); \mathrm{M2}\right)$ is more than two orders of magnitudes smaller than (16) and leads to $|\Im m\, M_d| = 0.018 \times 10^{-3}/m_{\pi^+}$, where $M_d \equiv M_{\pi^0 d}/4$. Nevertheless, the role of the M2 transition in the coherent production at threshold deserves a more detailed investigation which will be presented elsewhere. The main reasons are: (*i*) it provides an additional possibility to test theoretical predictions (an experimental separation of E1

and M2 requires a polarized deuteron target), and (*ii*) the relative importance of the M2 transition grows with the momentum transfer in the electroproduction process.

The result (17) has to be compared with the value calculated in [4],

$$E_d^{\chi PT} = 0.38 E_{0+}^{\pi^0 n} - 2.6 \times 10^{-3}/m_{\pi^+} = -1.8 \times 10^{-3}/m_{\pi^+}, \tag{18}$$

where the latter value is obtained taking the χPT prediction of [10] for the neutron electric dipole amplitude, $E_{0+}^{\pi^0 n} = 2.13 \times 10^{-3}/m_{\pi^+}$. Thus the threshold cross section itself is affected by less than 2% only by this imaginary part (17). However, there is no reason at all to assume that the contribution of the absorptive process to the real part of the amplitude is much smaller in magnitude than the imaginary part. Unfortunately, unitarity does not allow to estimate it. It can only be calculated within a model which to our knowledge has not yet been done. For the moment, in order to get a rough idea, one may look into the in many respects analogous situation for πd elastic scattering. There, three-body calculations suggest for the absorptive contribution to the scattering length, $\Re e\, a_{\pi d}^{abs} \approx -\Im m\, a_{\pi d}$ (see [5] for an overview). Assuming, therefore $\left|\Re e\, E_d^{abs}\right| = 0.22 \times 10^{-3}/m_{\pi^+}$, one has to conclude that the neglect of the absorption process in an analysis of the $\gamma d \to \pi^0 d$ cross section based on (18) would lead to a systematic error of the order of $\delta E_{0+}^{\pi^0 n} = \pm 0.6 \times 10^{-3}/m_{\pi^+}$ for the neutron amplitude. This rough argument at least demonstrates that a calculation of the absorptive contribution is necessary before definite conclusions on the neutron amplitude can be drawn. One way to do this is to combine conventional models for deuteron photodisintegration [8] with those for the pionic disintegration of the deuteron [11].

As second point we would like to address the question what is the role of this absorptive process in the framework of heavy baryon χPT. The χPT treatment of the process $\gamma d \to \pi^0 d$ is sketched in Fig. 1. The non-absorptive contribution, Fig. 1(a), is based on the two-nucleon irreducible kernel $K_{\gamma\pi}$ for the $\gamma pn \to \pi^0 pn$ subprocess. It is obtained from the effective chiral Lagrangian, and then sandwiched between deuteron wave functions ψ_d. $K_{\gamma\pi}$ sums all time-ordered diagrams which do not contain pure two-nucleon intermediate states. It has been calculated in [4] up to and including all terms of order $\nu \leq 0$ of the expansion in terms of

powers of small momenta $(Q/\Lambda)^\nu$ where typically $Q \sim m_\pi$, $\Lambda \sim m_N$.

The proper treatment of the absorptive contribution is shown in Fig. 1(b). Formally it is given by

$$T^{\chi PT/abs} = \psi_d^\dagger \, K_\pi G_0 K_\gamma \, \psi_d + \psi_d^\dagger \, K_\pi G_0 T_{NN}(W_0 + i\epsilon) G_0 K_\gamma \, \psi_d. \tag{19}$$

Here, the input from χPT are the two-nucleon irreducible kernels K_γ and K_π for $\gamma pn \to pn$ and $pn \to \pi^0 pn$, respectively. These are linked by either the free two-nucleon propagator,

$$G_0 = \left(W_0 - 2m_N - \vec{p}^{\,2}/m_N + i\epsilon\right)^{-1}, \tag{20}$$

or the propagation via $G_0 T_{NN} G_0$ which includes the nucleon-nucleon interaction through the full off-shell T-matrix in the ${}^3P_1$ partial wave. T_{NN} (and ψ_d) has to be calculated by solving the Lippmann-Schwinger equation for a two-nucleon potential. Ideally, the potential is thereby also taken from χPT as the sum of all irreducible $NN \to NN$ diagrams.

Now the question arises, where the absorptive contribution fits into the χPT power counting scheme. We will answer it by means of Fig. 2. The time-ordered graph of Fig. 2(a) shows an absorptive contribution which is contained in the first term of (19). The diagram of Fig. 2(b) is a part of $K_{\gamma\pi}$ build up from the same interaction vertices as (a). According to Weinberg's power counting rules [12–14], irreducible graphs with N nucleons, C separate connected pieces, L loops, and V_i vertices of type i contribute to order Q^ν with ν given by

$$\nu = 4 - N - 2C + 2L + \sum_i V_i \Delta_i, \tag{21}$$

where the index Δ_i is bounded by chiral invariance. For pure hadronic vertices one has $\Delta_i \geq 0$, while for vertices with one photon $\Delta_i \geq -1$. For the graph of Fig. 2(b) with $N = 2$, $C = L = 1$ this leads to $\nu_{2(b)} \geq 1$. Consequently these type of contributions could be ignored by Beane et al. [4] in order to be consistent up to and including $\nu \leq 0$.

However, as has been stressed by Weinberg, graphs with intermediate states containing *only* nucleons violate the simple power counting rules, because of the small (nearly infrared-divergent) energy denominators associated with the propagation of these states, see Eq.

(20). Fig. 2(a) belongs to this class of reducible graphs. In time-ordered perturbation theory, energy denominators of states with at least one pion are of order Q, while those with nucleons only are of order Q^2/m_N, when one assumes that all momenta are of order Q. This leads to count the order of Fig. 2(a) as $\nu_{2(a)} = \nu_{2(b)} - 1 \geq 0$.

It could be objected, that the momenta of the intermediate particles in Fig. 2(a) are not of order Q but rather of order $P \sim \sqrt{m_\pi m_N}$ which implies that the two-nucleon energy denominator is of order Q^{-1} rather than Q^{-2}. Actually this is true for the imaginary part of the diagram which just arises from the kinematical situation where the nucleon momenta are equal to the on-shell momentum p_{pn} which is given by $p_{pn}^2/m_N = m_\pi$ applying nonrelativistic kinematics. In such a situation also the two intermediate pion momenta must be of order P in order to change the relative nucleon momenta of order Q, provided by the deuteron wave function. However, as will be seen soon, counting all powers of P in the graph (a) of Fig. 2 will not change the above conclusion that it contributes already to the order Q^0. The necessity to consider a modified power counting (due to the kinematics of the reaction) was first noted by Cohen et al. [15], studying the reaction $pp \to \pi^0 pp$ near threshold. In order to get the worst case, we assume from now on that all vertices in Fig. 2(a) arise always from the leading terms of the chiral Lagrangian. Following the steps in [12] one counts for Fig. 2(a): P^3 from three derivative couplings, P^{-2} from the two energy denominators of the states containing pions, P^{-2} from four factors $1/\sqrt{2E_\pi}$, a factor P^3 from the integral over the loop three-momentum, and finally P^{-2} from the two-nucleon energy denominator. Altogether, the graph (a) of Fig. 2 gives a contribution of the order P^0 or equivalent Q^0.

Thus we have to conclude that a complete calculation including all terms of order Q^ν with $\nu \leq 0$ requires the inclusion of the absorptive process. Indeed, already the imaginary contribution of the absorption process, $\Im m\, E_d$ in (17), turns out to be of the same order of magnitude as the three-body contribution of order $\nu = 0$ calculated by Beane et al., $E_d^{tb,4} = -0.25 \times 10^{-3}/m_{\pi^+}$ in the notation of [4].

In summary, the contribution of the two-step process $\gamma d \to pn \to \pi^0 d$ to the imaginary part of the electric dipole (and magnetic quadrupole) amplitude for coherent pion photo-

production on the deuteron has been calculated utilizing unitarity constraints. The result shows that this absorptive process cannot be neglected in the extraction of the elementary neutron amplitude. It has been shown that a consistent χPT calculation for $\gamma d \to \pi^0 d$ beyond next-to-leading order requires indeed the inclusion of the absorptive process.

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FIGURES

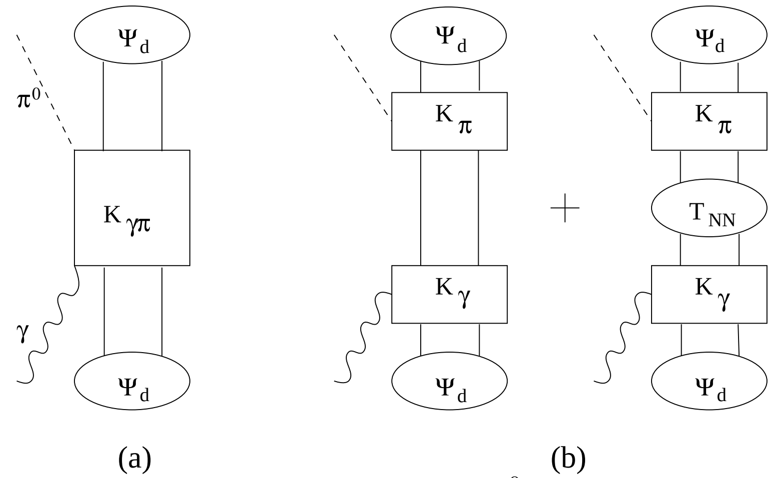

FIG. 1. Schematical representation of the $\gamma d \to \pi^0 d$ amplitude at threshold in the χPT framework: the non-absorptive part (a) is based on the irreducible $\gamma pn \to \pi^0 pn$ kernel $K_{\gamma\pi}$, and the absorptive part (b) combines the irreducible kernels K_γ and K_π for $\gamma pn \to pn$ and $pn \to \pi^0 pn$, respectively. ψ_d is the deuteron wave function and T_{NN} the nucleon-nucleon scattering matrix in the $^3P_1$ partial wave.

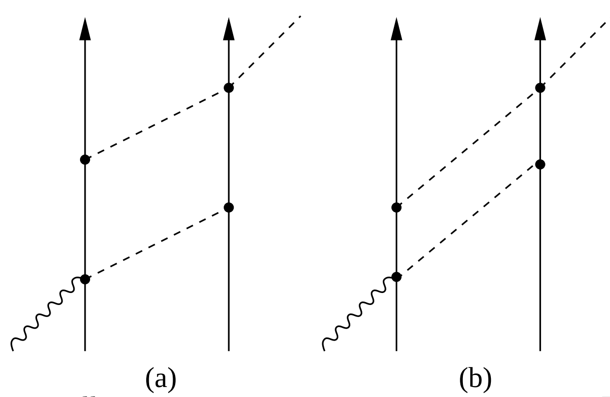

FIG. 2. Two diagrams to illustrate our power counting arguments. The time-ordered diagram (a) is part of the absorptive process, whereas diagram (b) although built up from the same vertices contributes to the non-absorptive part due to a different time-ordering. Solid, dashed, and wavy lines represent nucleons, pions, and photons, respectively.